On Computing the Hamiltonian Index of Graphs

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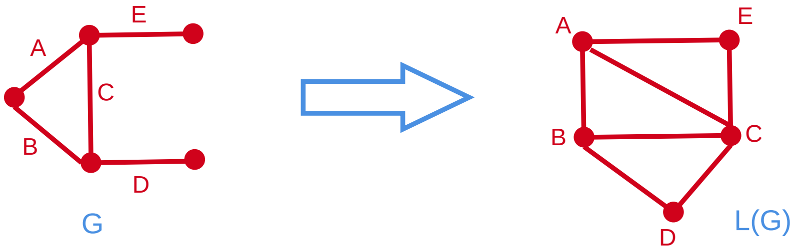
Outline

Line Graph

- The line graph of an undirected graph G is another graph $L(G)$ that represents the adjacencies between edges of G.
- The vertex set of the *line graph* of a graph G—denoted $L(G)$—is the edge set $E(G)$ of G, and two vertices e, f are adjacent in $L(G)$ if and only if the edges e and f share a vertex in G.

Example

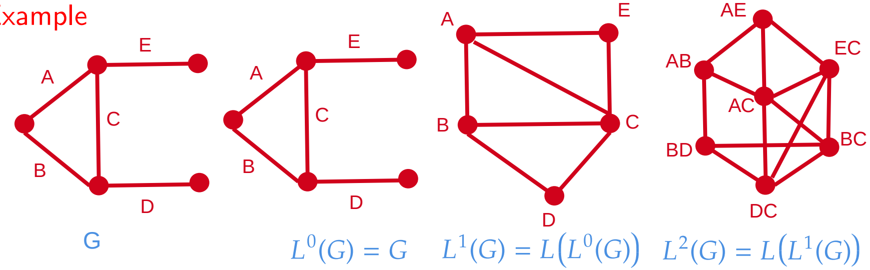


Iterated Line Graph

- For an integer $r \geq 0$ the *r- th iterated line graph* $L^r(G)$ of a graph G is defined by:
  - $L^0(G) = G$ and
  - $L^r(G) = L(L^{(r-1)}(G))$ for $r > 0$,

  where $L(G)$ denotes the line graph of G.
- If G is a connected graph which is *not* a path then $L^r(G)$ is nonempty for *all* $r \geq 0$ [CJIS90].

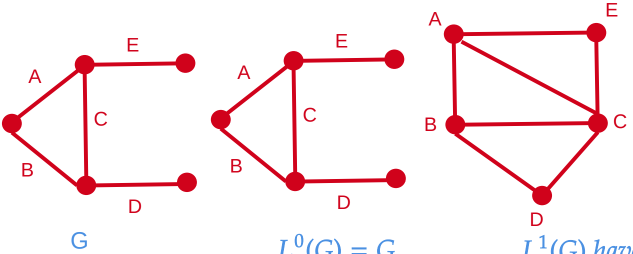
Example



Hamiltonian Index

- The *Hamiltonian Index* $h(G)$ of G is the smallest r such that $L^r(G)$ has a Hamiltonian cycle [Cha68a].

Example



$L^0(G) = G$ *does not have a HC*

$L^1(G)$ *have a HC*

$h(G) = 1$

Computing Hamiltonian Index Contd....

- Checking if $h(G) = 0$ holds $\equiv$ checking if graph G is Hamiltonian
  1. long known to be NP-complete.
  2. even when the input graph is planar and *subcubic*[GJT76].
- Checking if $h(G) = 1$ holds $\equiv$ checking if
  1. G is *not* Hamiltonian, and
  2. the line graph $L(G)$ is Hamiltonian.
  3. problem is NP-complete [Ber81] even for subcubic graphs [RWX11].

Computing Hamiltonian Index Contd....

- Checking if the line graph $L(G)$ of a graph G is Hamiltonian are equivalent to the following :
  1. G has an edge Hamiltonian cycle [Cha68a]
  2. G contains a closed trail T such that every edge in G has at least one end-point in T [HNW65]
- An *edge Hamiltonian path* of a graph G is any permutation Π of the edge set $E(G)$ of G such that every pair of consecutive edges in Π has a vertex in common.
- An *edge Hamiltonian cycle* of G is an edge Hamiltonian path of G in which the first and last edges also have a vertex in common.

Hamiltonian Index Contd....

- If the minimum degree of a graph G is at least three then $h(G) \leq 2$ holds[CW73].
- Checking whether $h(G) = t$ is NP-complete — for *any* fixed integer $t \geq 0$, even when the input graph G is subcubic.
- Goal - *Parameterized complexity analysis of the problem of computing the Hamiltonian Index.*

Parameterized Complexity

- One of the ways to deal with NP-hard problems.
- An instance of a *parameterized problem* is a pair (x, k) where x is an instance of a classical problem and k is a (usually numerical) *parameter* which captures some aspect of x.
- FPT: A problem is *fixed parameter tractable* (FPT) with respect to parameter k if there exists an algorithm running in $f(k).n^{O(1)}$ time - this running time is abbreviated as $\mathcal{O}^{\star}(f(k))$.

Problem Definition

HAMILTONIAN INDEX(HI) **Parameter:** tw

Input: A connected undirected graph $G = (V, E)$ which is not a path, a tree decomposition $\mathcal{T} = (T, \{X_t\}_{t \in V(T)})$ of G of width tw, and $r \in \mathbb{N}$.

Question: Is $h(G) \leq r$?

- **Our Result** - The HAMILTONIAN INDEX problem is fixed-parameter tractable – there is an algorithm which solves an instance $(G, \mathcal{T}, tw, r)$ of Hamiltonian Index in $\mathcal{O}^\star((1 + 2^{(\omega+3)})^{tw})$ time, where ω denotes the matrix multiplication exponent ($\omega < 2.3727$).

Related Work

- The parameterized complexity of computing $h(G)$ — has not been previously explored.
- Two special cases of checking if $h(G) \in \{0, 1\}$ — have been studied with the treewidth tw of the input graph G as the parameter
  1. Checking whether $h(G) = 0$ holds — i.e., whether G is Hamiltonian
     - was long known to be solvable in $\mathcal{O}^\star(tw^{\mathcal{O}(tw)})$ time.
     - In 2011—-showed that this can be done in randomized $\mathcal{O}^\star(4^{tw})$ time [CNP+11].
     - Bodlaender et al.[BCKN15] and Fomin et al. [FLPS16]- showed that this can be done in *deterministic* $\mathcal{O}^\star(2^{\mathcal{O}(tw)})$ time.

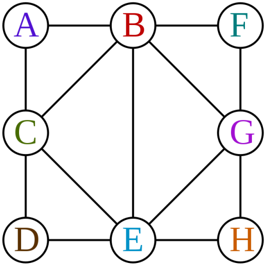
Related Work

- Checking whether $h(G) = 1$ holds — i.e. whether L(G) is hamiltonian
  - - first addressed indirectly by Lampis et al[LMMU17].
    - addressed Edge Hamiltonian Cycle(EHC) and Edge Hamiltonian Path(EHP) problems.
    - showed that EHP is FPT if and only if EHC is FPT, and that these problems can be solved in $\mathcal{O}^\star(tw^{\mathcal{O}(tw)})$ time.
  - - Misra et al. investigated an optimization variant of Edge Hamiltonian Path - called LONGEST EDGE-LINKED PATH (LELP).
    - An *edge-linked path* is a sequence of edges in which every consecutive pair has a vertex in common.
    - Given a graph G, $k \in \mathbb{N}$, and a tree decomposition $\mathcal{T}$of G of width tw as input the LELP problem asks whether G has an edge-linked path of length at least k.
    - Setting $k = |E(G)|$ yields EHP as a special case.
    - solved LELP (and hence, EHPand EHC) in $\mathcal{O}^\star((1 + 2^{(\omega+3)})^{tw})$ time.

Tree decomposition and Treewidth

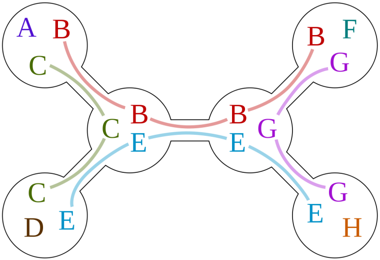
- A *tree decomposition* of a graph G is a pair $\mathcal{T} = (T, \{X_t\}_{t \in V(T)})$ where T is a tree and every vertex t of T is assigned a subset $X_t \subseteq V(G)$ of the vertex set of G.
- Each X_t - called a *bag*,

Example



A graph G

$\Longrightarrow$



A tree decomposition of G

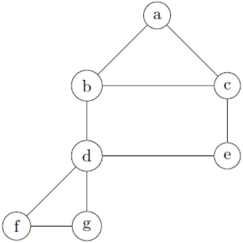
Tree decomposition and Treewidth

- The structure satisfies the following conditions:
  1. Every vertex of G is in at least one bag.
  2. For every edge uv in G there is at least one node $t \in V(T)$ such that $\{u, v\} \subseteq X_t$.
  3. For each vertex v in G the set $\{t \in V(T)\ ;\ v \in X_t\}$ of all nodes whose bags contain v, form a *connected subgraph* (i.e, a sub-tree) of T.
- The *width* of tree decomposition - maximum size of a bag, minus one.
- The *treewidth* of a graph G, denoted $tw(G)$, - minimum width of a tree decomposition of G.

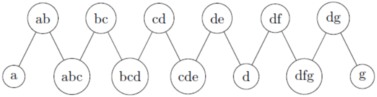
Nice Tree Decomposition

- A *nice tree decomposition* of a graph G is a tree decomposition $\mathcal{T} = (T, \{X_t\}_{t \in V(T)})$ with the following additional structure:
  1. The tree T is *rooted* at a distinguished *root* node $r \in V(T)$.
  2. The bags associated with the root node r and with every leaf node are all empty.

Example



A graph G



A nice tree decomposition of G

Nice Tree Decomposition Contd...

- Every non-leaf node is one of four types:
  1. *Introduce vertex node*: — a node $t \in V(T)$ with exactly one child node t' such that $(X_t \setminus X_{t'}) = \{v\}$ for some vertex $v \in V(G)$–i.e., *the vertex v is introduced at node t*.
  2. *Introduce edge node*: — a node $t \in V(T)$ with exactly one child node t' such that $X_t = X_{t'}$. Further, the node t is labelled with an edge $uv \in E(G)$ such that $\{u, v\} \subseteq X_t$; the edge uv is *introduced* at node t. *Every edge in the graph G is introduced at exactly one introduce edge node in the entire tree decomposition*.
  3. *Forget* node: — a node $t \in V(T)$ with exactly one child node t' such that $(X_{t'} \setminus X_t) = \{v\}$ for some vertex $v \in V(G)$; the vertex v is *forgotten* at node t.
  4. *Join* node: — a node $t \in V(T)$ with exactly two child nodes t_1, t_2 such that $X_t = X_{t_1} = X_{t_2}$.

Nice Tree Decomposition Contd.....

- For a node $t \in V(T)$ of the nice tree decomposition $\mathcal{T}$, let
  1. T_t — the subtree of T which is rooted at t.
  2. V_t — the union of all the bags associated with nodes in T_t.
  3. E_t — the set of all edges introduced in T_t, and
  4. $G_t = (V_t, E_t)$ – *the subgraph of G defined by* T_t.

Eulerian Steiner Subgraph

- A key part of our algorithm for computing $h(G)$ consists of solving Eulerian Steiner Subgraph:

EULERIAN STEINER SUBGRAPH(ESS) **Parameter:** *tw*
Input: An undirected graph $G = (V, E)$, a set of "terminal" vertice $K \subseteq V$, and a tree decomposition $\mathcal{T} = (T, \{X_t\}_{t \in V(T)})$ of G, of widt tw.
Question: Does there exist an Eulerian subgraph $G' = (V', E')$ of such that $K \subseteq V'$?

- Eulerian Steiner Subgraph problem is NP-complete[Pul79].

An FPT algorithm for Eulerian Steiner Subgraph

- Input: an instance $(G, K, \mathcal{T}, tw)$ of Eulerian Steiner Subgraph
- Our algorithm tells in $\mathcal{O}^\star((1 + 2^{(\omega+3)})^{tw})$ time whether graph G has a subgraph which is
  1. Eulerian, and
  2. contains every vertex in the terminal set K.
- Obtain a *nice* tree decomposition from $\mathcal{T}$ in polynomial time [Klo94].
- Perform dynamic programming (DP) over the bags of this nice tree decomposition.
- Pick an arbitrary terminal $v^\star \in K$ and add it to every bag of $\mathcal{T}$.
- Let $\mathcal{T}$ refer to the resulting "nearly-nice" tree decomposition in which the bags at all the leaves and the root are equal to $\{v^\star\}$.

An FPT algorithm for Eulerian Steiner Subgraph

- How an Eulerian subgraph $G' = (V', E')$ of G which contains all the terminals K interacts with the structures defined by node t in the following way
- For a bag X_t and subsets $X \subseteq X_t$, $O \subseteq X$, we say that a partition $P = \{X^1, X^2, \dots X^p\}$ of X is *valid for the combination* (t, X, O) if there exists a subgraph $G'_t = (V'_t, E'_t)$ of G_t such that
  1. $X_t \cap V(G'_t) = X$.
  2. G'_t has exactly p connected components $C_1, C_2, \dots, C_p$ and for each $i \in \{1, 2, \dots, p\}$, $X^i \subseteq V(C_i)$.
  3. Every terminal vertex from $K \cap V_t$ is in $V(G'_t)$.
  4. The set of odd-degree vertices in G'_t is exactly the set O.
- Such a subgraph G'_t of G_t - *is a witness for* $((t, X, O), P)$.

An FPT algorithm for Eulerian Steiner Subgraph

- For each node t of the tree decomposition $\mathcal{T}$ compute the set of all partitions P which are valid for the combination (t, X, O).
- At the root node r the algorithm would apply validity condition to decide the instance $(G, K, \mathcal{T}, tw)$.
- The running time of this algorithm could have a factor of tw^{tw}.
- To avoid this after computing a set $\mathcal{A}$ of valid partitions for each combination (t, X, O) we compute a representative subset $\mathcal{B} \subseteq \mathcal{A}$ and throw away the remaining partitions $\mathcal{A} \setminus \mathcal{B}$.
- Thus the number of partitions which we need to remember for any combination (t, X, O) never exceeds 2^{tw}.
- The entire DP can be done in $\mathcal{O}^{\star}((1 + 2^{(\omega+3)})^{tw})$ time.

Finding the Hamiltonian Index

- Input: An instance $(G, \mathcal{T}, tw, r)$ of Hamiltonian Index.
- Output: In $\mathcal{O}^{\star}((1+2^{(\omega+3)})^{tw})$ time outputs whether graph G has Hamiltonian Index at most r.
- If G is a connected graph on n vertices which is not a path, then $L^r(G)$ is Hamiltonian for all integers $r \geq (n-3)$ [Cha68a].
- If $r \geq (|V(G)| - 3)$ holds then our algorithm returns **yes**.
- If $r < (|V(G)| - 3)$ then it checks, for each $i = 0, 1, \ldots, r$ in increasing order, whether $h(G) = i$ holds.

Finding the Hamiltonian Index

- How we check if $h(G) = i$ holds for increasing values of i.
  1. For $i = 0$ we apply an algorithm of Bodlaender et al[BCKN15].
  2. For $i = 1$ we apply a classical result of Harary and Nash-Williams[HNW65].
  3. For checking if $h(G) \in \{2, 3\}$ holds we make use of a structural result of Hong et al. [HLTC09].
     - Let G be a connected graph with $h(G) \geq 2$ and with at least one vertex of degree at least three, and let $\tilde{H}^{(2)}, \tilde{H}^{(3)}$ be graphs constructed from G using [HLTC09]. Then
     - $h(G) = 2$ if and only if $\tilde{H}^{(2)}$ has a spanning Eulerian subgraph; and
     - $h(G) = 3$ if and only if $h(G) \neq 2$ and $\tilde{H}^{(3)}$ has a spanning Eulerian subgraph.
  4. For checking if $h(G) = i$ holds for $i \in \{4, 5, \dots\}$ we use a reduction due to Xiong and Liu [XL02].
     - If G is a connected graph with $h(G) \geq 4$, then using [XL02], we construct a graph $h'(G)$ such that $h(G) = h'(G) + 1$.

Finding the Hamiltonian Index

- Checking Hamiltonicity takes $\mathcal{O}^\star((5 + 2^{(\omega+2)/2})^{tw})$ time.
- Checking if $L(G)$ is Hamiltonian takes $\mathcal{O}^\star((1 + 2^{(\omega+3)})^{tw})$ time.
- The graphs $\tilde{H}^{(2)}$ and $\tilde{H}^{(3)}$ can each be constructed in polynomial time, and checking if each has a spanning Eulerian subgraph takes $\mathcal{O}^\star((1 + 2^{(\omega+3)})^{tw})$ time.
- The running time of the algorithm satisfies the recurrence $T(r) = \mathcal{O}^\star((1 + 2^{(\omega+3)})^{tw}) + T(r-1)$.
- The recurrence resolves to $T(r) = \mathcal{O}^\star((1 + 2^{(\omega+3)})^{tw})$.

Conclusion

- The Hamiltonian Index $h(G)$ - introduced by Chartrand in 1968.
- Checking if $h(G) = t$ holds is NP-hardfor any fixed integer $t \geq 0$.
- the problem is FPTand - $\mathcal{O}^{\star}((1 + 2^{(\omega+3)})^{tw(G)})$ time.
- This running time matches that of the current fastest algorithm, due to Misra et al. [MPS19], for checking if $h(G) = 1$ holds.
- Eulerian Steiner Subgraphproblem - $\mathcal{O}^{\star}((1 + 2^{(\omega+3)})^{tw(G)})$ time.
- Our result on Eulerian Steiner Subgraph could turn out to be useful for solving other problems as well.
- Whether there exists a matching lower bound, or can this be improved?
- Can $h(G)$ be found in the same FPT running time as it takes to check if G is Hamiltonian?

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